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A MODEL BASED ANALYSIS OF DYNAMICS OF A SINGLE PINION ELECTRIC POWER STEERING SYSTEM

Summary: An analysis of dynamics for an electric power assisted steering system which is modelled as a four mass electromechanical system with the main focus on the mechanical part. The subject of the paper is to create a model which may find application in the steer-by-wire technology to mimic steering feel of a mechanically coupled system.

Keywords: electric power steering, analysis of dynamics, a mimic model for steer-by-wire

ANALIZA DYNAMIKI ELEKTRYCZNIE WSPOMAGANEGO SYSTEMU KIEROWNICZEGO Z ZASTOSOWANIEM MODELOWANIA

Streszczenie: W artykule opisano analizę dynamiczną systemu kierowniczego pojazdów. Przedmiotem badań był system kierowniczy wspomagany elektrycznie. Ten system modelowano jako system elektro-mechaniczny o czterech masach, w którym najwięcej uwagi poświęcono części mechanicznej. Przedmiotem artykułu jest zbudowanie modelu, który może być stosowany w wirtualnych analizach systemów kierowniczych (wspomaganych elektrycznie). Zbudowany model można uznać za mimetyczny, czyli adekwatnie oddający cechy, zatem nadaje się do analiz złożonych systemów mechanicznych.

Keywords: układ kierowniczy z zasilaniem elektrycznym, analiza dynamiki

1. Introduction

Steering systems are the key safety systems which are installed in the present vehicles. It should be emphasized that it is the most important safety system in the vehicle, even above the braking system. A supporting argument for this statement is that its functionality is not redundant to at least some extent. For instance, a broken braking system can be in some extent duplicated by a hand brake or engine; both listed systems allow reduce the vehicle's speed. On the other side, with a malfunctioning or broken steering systems no other options are available to steer the vehicle.

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1.1. Electric power steering

Electric power steering systems are an evolution of previously widely spread hydraulic steering systems and electro-hydraulic steering systems. EPS[2] Systems have a series of advantages over their predecessors. EPS reaction for a control command is faster than in a hydraulic system because it is performed directly by an electric motor instead increasing/decreasing fluid pressure in the system. An additional advantage is that EPS does not require a car's engine to be running to maintain hydraulic pressure. Furthermore, a majority of energy is consumed by EPS only when it is required to provide an assistance torque to support a driver, thus, its overall carbon footprint is smaller in contrast to its predecessors.

1.2. Types of electric power steering systems

Nowadays on the market a three main types of the EPS can be distinguished:

Table 1. Main types of electric power steering

Types	Sub-types
Column EPS (C-EPS)	n/a
Pinion EPS (P-EPS)	Single pinion (SP-EPS)
	Dual pinion (DP-EPS)
Rack EPS (R-EPS)	Belt drive
	Coaxial drive

Depending on the vehicle size and a car manufacturer allocation of vehicle's components in the engine compartment different types of electric power steering find their possible application. In general, column type and single pinion EPS are designated to cars from segments A, B or C. Dual pinion and rack EPS are recommended by EPS manufactures for cars from higher segments and small trucks, for instance Ford F-150 or Dodge Ram.

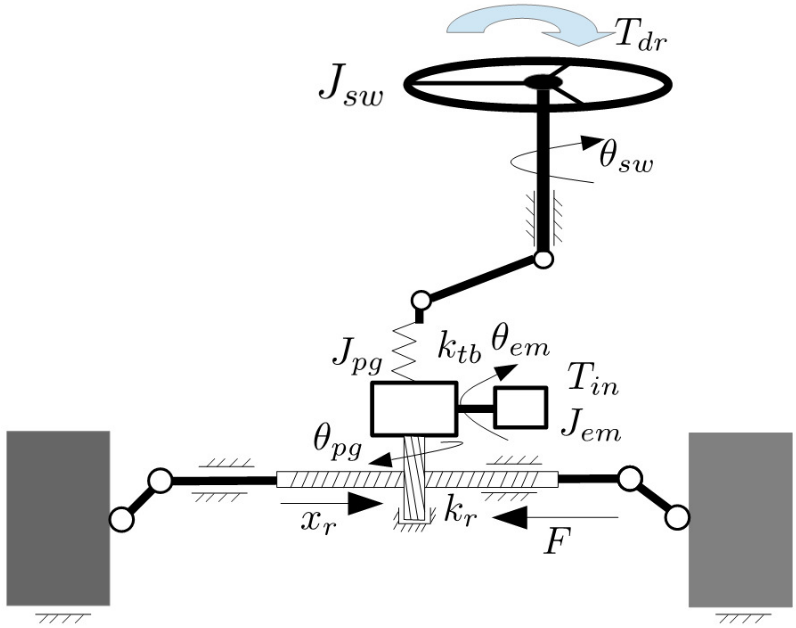
1.3. Present and future applications of models

Present electric power steering is a sophisticated device which is equipped with a specialized embedded computer. EPS beside of its dominant function, i.e. to provide the assistance torque to a driver, have some other features like for instance: Lane Keeping Assist (LKA), Parking Assist, Haptic Feedback and much more. Highly skilled and experienced drivers, who are employed by the car manufactures (OEMs), spend hours on test tracks and public roads to tune electric power steering with regard to safety and steering feel. Both mentioned aspects are pivotal especially with upcoming steer-by-wire technology where mechanical coupling between the driver and the rack is to be removed, however, this decoupling has to be reflected on the steering wheel to provide the driver the same steering feel as if mechanical coupling had existed.

[2] EPS – Electric Power Steering

2. The considered single pinion EPS model

In order to perform analysis of a single pinion EPS a model was created. A set of research publications as well as technical specifications were read. The received outcome of the literature analysis allowed to supply the simulation model with data which is close to the real system. Unfortunately not all needed data was found and some engineering assumptions had to be undertaken. For instance, as it is clearly visible on the picture on the next page, which is a sketch of the SP-EPS model, there is a double universal joint in the system. It was assumed that this double universal joint is fully compensated, as a result of its bending angles selection, and all possible effects of the a not compensated universal joint are skipped in the further considerations.



Picture 1. A conceptual model of single pinion EPS

2.1. A mathematical model of SP-EPS

A formalized representation of the mathematical model:

$$\boldsymbol{A} \cdot \ddot{\overline{\xi}} + \boldsymbol{B} \cdot \dot{\overline{\xi}} + \boldsymbol{C} \cdot \overline{\xi} = \overline{q} \tag{1}$$

where:
$\boldsymbol{A}$ − *matrix of masses and moments of interia*
$\boldsymbol{B}$ − *matrix of viscous damping constants*
$\boldsymbol{C}$ − *matrix of stifnesses*
$\ddot{\overline{\xi}}$ − *vector of linear and angular accelerations*

$\dot{\bar{\xi}}$ $-$ *vector of linear and angular velocities*
$\bar{\xi}$ $-$ *vector of linear and angular displacements*
$\bar{q}$ $-$ *vector of external forces and torques*

$$\begin{bmatrix} J_{sw} & 0 & 0 & 0 \\ 0 & J_{pg} & 0 & 0 \\ 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & J_{em} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_{sw} \\ \ddot{\theta}_{pg} \\ \ddot{x}_r \\ \ddot{\theta}_{em} \end{bmatrix} + \begin{bmatrix} h_{sw} & 0 & 0 & 0 \\ 0 & h_{pg} & 0 & 0 \\ 0 & 0 & h_r & 0 \\ 0 & 0 & 0 & h_{em} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_{sw} \\ \dot{\theta}_{pg} \\ \dot{x}_r \\ \dot{\theta}_{em} \end{bmatrix} + \begin{bmatrix} k_{tb} & -k_{tb} & 0 & 0 \\ -k_{tb} & k_{tb} + k_r & -\frac{k_r}{r} & 0 \\ 0 & -\frac{k_r}{r} & \frac{k_r}{r^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \theta_{sw} \\ \theta_{pg} \\ x_r \\ \theta_{em} \end{bmatrix} = \begin{bmatrix} T_{dr} \\ \eta \cdot i \cdot T_{in} \\ F \\ T_{em} - T_{in} \end{bmatrix} \quad (2)$$

Table 2. Model parameters and main variables

Magnitude	Value and unit	Description
θ_{sw}	$[°]$	Steering wheel angle
J_{sw}	$3 \cdot 10^{-2} [kg \cdot m^2]$	Steering wheel moment of inertia
h_{sw}	$0.072\ [N \cdot s \cdot rad^{-1}]$	Steering wheel viscous damping
k_{tb}	$2.6\ \left[\frac{N \cdot m}{°}\right]$	Torsion bar stiffness
θ_{pg}	$[°]$	Pinion-gearbox assembly angle
J_{pg}	$0.5\ [kg \cdot m^2]$	Pinion-gearbox assembly moment of inertia
h_{pg}	$0.5\ [N \cdot s \cdot rad^{-1}]$	Pinion-gearbox assembly viscous damping
r	$55\ \left[\frac{mm}{rev}\right]$	Pinion-rack C factor
i	$24\ [-]$	Gearbox ratio
η	$0.8\ [-]$	Worm gearbox efficiency
x_r	$[mm]$	Rack ‘s linear displacement
m_r	$1000\ [kg]$	Collective mass of vehicle front, tires on the rack
k_r	$4000\ \left[\frac{N \cdot m}{rad}\right]$	Transformed by pinion-rack C-factor vehicle’s front suspension and rack’s stiffness
h_r	$35 \cdot 10^3\ [N \cdot s \cdot m^{-1}]$	Rack’s viscous damping
θ_{em}	$[°]$	Electric motor’s shaft angle
J_{em}	$76 \cdot 10^{-4} [kg \cdot m^2]$	Electric motor ‘s moment of inertia
h_{em}	$0.05\ [N \cdot s \cdot rad^{-1}]$	Electric motor’s viscous damping
T_{em}	$4.5..6\ [N \cdot m]$	Electromagnetic torque
T_{in}	$\leq 4.5\ [N \cdot m]$	Assistance torque from electric motor
T_{dr}	$10 [N \cdot m]$	Driver’s torque on the steering wheel
F	$10.5\ [kN]$	Force on the rack

2.2. Simulation algorithm in GNU Octave

The presented mathematical model of the single pinion electric power steering has been implemented in the GNU Octave environment in order to solve it numerically with application of Runge-Kutta method of the second order. A set of script was created to perform simulation.
A piece of the GNU Octave script to invoke the numerical analysis

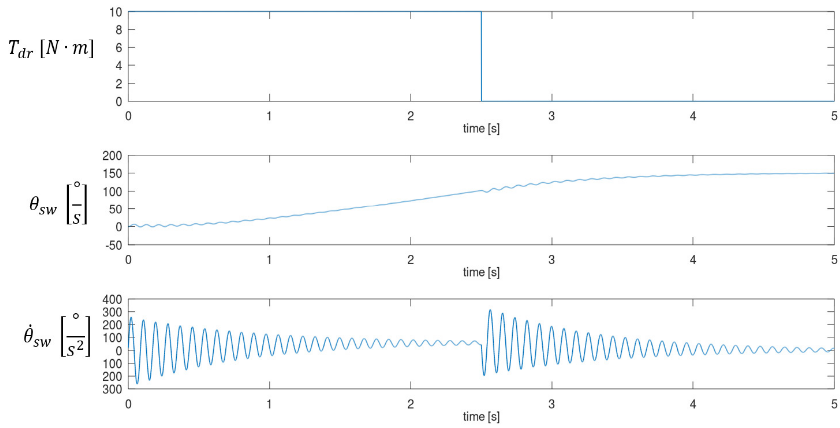
```
for j=1:length(t)

    Tdr = Tdrt(j);
    Tin = Tint(j);
    F   = Fex(j);

    currSample = j;
    z = diffequ(@singlePinionEPS, t(j), z, 2);
    Osw(j)  = radToDegFac * z(1);
    dOsw(j) = radToDegFac * z(2);
    Opg(j)  = radToDegFac * z(3);
    dOpg(j) = radToDegFac * z(4);
    xr(j) = z(7);
    dxr(j) = z(8);

end
```

A function script which implements a numerical form of the SP-EPS mathematical model:

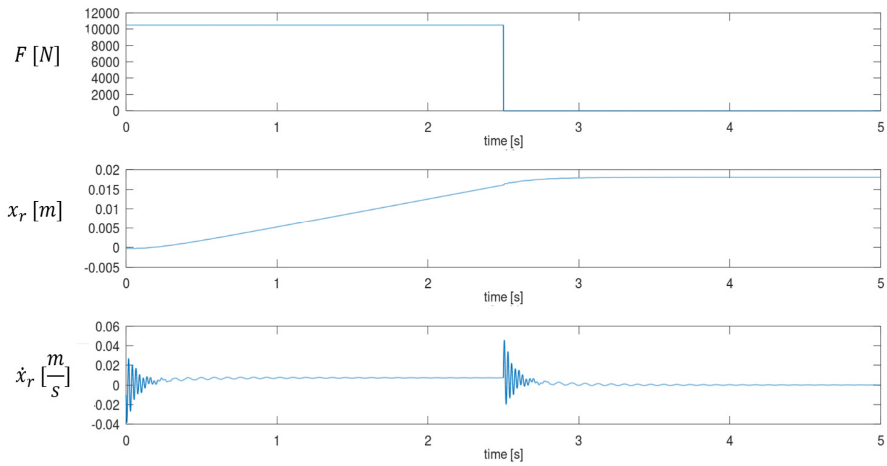
```
function [dz] = singlePinionEPS (t, z)

global Jsw Jpg Jem mr;
global hsw hpg hem hr;
global ktb kr r i n dp;
global Tdr Tin Tem F;

dz=zeros(8,1);

# angular velocity of a steering wheel
dz(1) = z(2);
# angular acceleration a steering wheel
dz(2) = 1/Jsw * (Tdr - ktb*(z(1) - z(3)) - hsw*z(2));
# angluar velocity of a pinion-gearbox assembly
dz(3) = z(4);
# angular acceleration of a pinion-gearbox assembly
dz(4) = 1/Jpg * (ktb*(z(1) - z(3)) + n*i*Tin - kr * (z(3) -
(z(7)/r)) - hpg*z(4));
# electric motor's shaft angular velocity
dz(5) = z(6);
# electric motor's shaft angular acceleration
dz(6) = 1/Jem * (Tem - hem * z(6) - Tin);
# rack's linear velocity
dz(7) = z(8);
# rack's linear acceleration
dz(8) = 1/mr * (kr/r * (z(3) - (z(7)/r)) - hr * z(8) - F)

endfunction
```

2.3. The considered model step response

A simulation of the model's dynamics has been performed by examining its step response, thereby, a driver torques in form of a pulse signal has been added to the steering wheel. An important assumption was made i.e. the simulated vehicle was not moving, therefore, the load on the rack in form of the force F. Additionally, a delay from measurement instruments and microcomputer has been introduced by means of adding the assistance torque after two milliseconds. This is presented on picture 4.

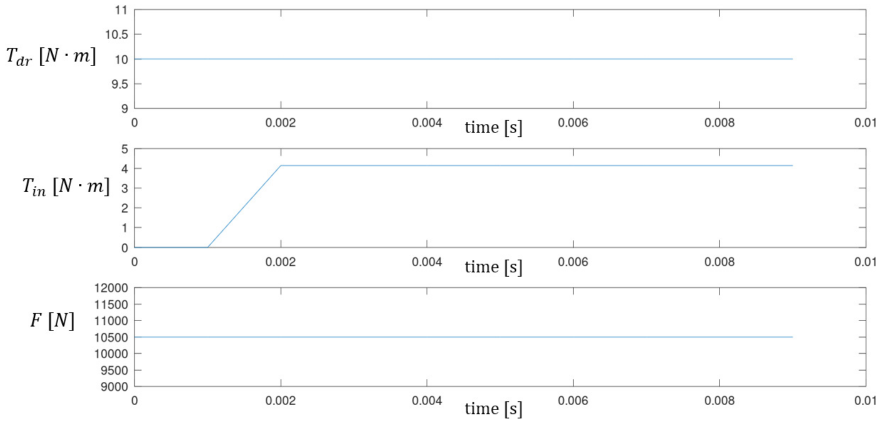


Picture 2. A driver's torque and the steering wheel angle's and angular velocity



Picture 3. A force on rack and the rack's linear displacement and linear velocity

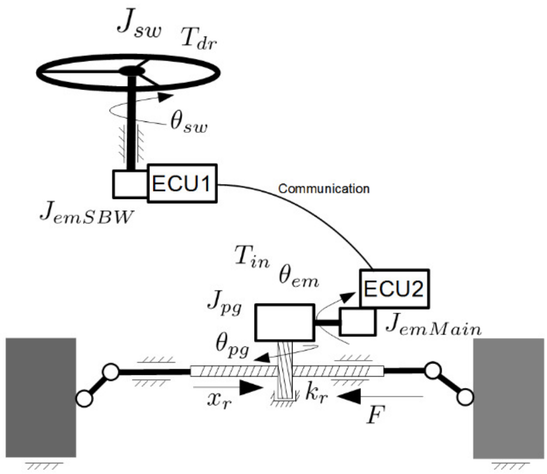
Vibrations can be spotted on the picture 2 which present a trace of the steering wheel angle. These vibrations are one of the main feedbacks to the driver from the vehicle. A future challenge with regard to steer by wire, in short sbw, is to mimic exactly the same feeling on the steering wheel as it is sensed in the conventional vehicles.



Picture 4. A delayed signal of the assistance torque (note: simulation step equal 1ms)

2.4. Steer-by-wire overview

Nowdays in automotive industry a research and development takes place to construct and introduce globally a sbw[3] electric power steering systems. A lot of work needs to be done with main focus on the safety aspects by means of meeting required FIT[4] levels.



Picture 4. A sketch of the steer by wire EPS system

[3] swb – steer-by-wire (also known as drive-by-wire)
[4] FIT – failure in time (1 FIT = one failure in 10^9 device hours)

3. Summary

The model of the single pinion electric power steering system presented in this paper describes one of possible approaches on the long road related to steer by wire technology. As it is symbolically presented on the previous page, the steer by wire system will be composed of two electrical motors. Main electrical motor is responsible to drive the rack according to the driver's commands and an auxiliary electrical motor which is responsible to provide to the driver an emulation feedback from the road. This means that a feedback's electric motor controller will implement the mathematical model to mimic desired steering feel. Additionally, an astute reader could spotted that a torsion bar is not installed in the proposed steer by wire EPS. This is a possible reduction of elements in the system by means of so called sensorless approach with regard to electric motors control.

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