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DOBÓR I UZASADNIENIE MATEMATYCZNEGO MODELU SYGNAŁU MOWY DO REHABILITACJI OSÓB Z WADAMI SŁUCHU

Streszczenie: Praca koncentruje się na problematyce modelowania matematycznego sygnałów mowy dla biotechnicznego systemu przywracania zdolności mówienia. Adekwatność modelu matematycznego sygnału mowy w postaci okresowo skorelowanego procesu losowego dla problemu korekcji mowy osób niesłyszących opiera się na danych eksperymentalnych. Opracowanie metody korelacji widmowej dla wyznaczania cech informatycznych sygnałów mowy z wykorzystaniem metod obliczeniowych z zastosowaniem okienek w celu uzyskania wartości składowych widmowych.

Słowa kluczowe: sygnał mowy, model matematyczny, okresowo skorelowany proces losowy, składowe spektralne, struktura systemu rehabilitacji biotechnicznej, stabilność

SELECTION AND GROUNDING OF A SPEECH SIGNAL MATHEMATICAL MODEL FOR REHABILITATION OF PEOPLE WITH DAMAGED HEARING SPEAKING ABILITIES

Summary: The work focuses on the problems of speech signals mathematical modelling for biotechnical system of speaking abilities rehabilitation. The adequacy of the speech signal mathematical model in the form of periodically correlated stochastic process for the problem of deaf people speech correction is grounded on the basis of experimental data. The spectral-correlation method for speech signals informative features determination using the calculating methods with windows application to obtain the spectral components values is developed.

Keywords: speech signal, mathematical model, periodically correlated stochastic process, spectral components, biotechnical rehabilitation system structure, stability

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1. Introduction

Selection and substantiation of a mathematical model of the speech signal, which adequately describes the patterns in the formation of speech sounds, ensures the effective functioning of the system designed for training (rehabilitation) of the speech apparatus of people with damaged hearing speaking abilities.

When teaching the language of people with damaged hearing speaking abilities, a method based on the use of sight is common (visual analyzer system), was described in the works of A.G. Zikeev [1], G. Pickett [2], H. Levitt [3], E.P. Kuzmicheva [4], M.Ya Kozlov., A.L. Levin [5] and others. But the use of such an approach is currently not effective enough, as the visual learning image reflects only the time-varying instantaneous features of the speech signal. It is obvious that in the known systems when modeling the speech signal does not take into account all the specifics of the model during the formation of the educational image required for the patient to control their speech apparatus. When rehabilitating the functions of the speech apparatus, it is necessary to take into account that the patient, using the data of the speech signal obtained through his visual organ, must reflexively approximate the visual images of the characteristics of his speech signal and the desired (standard). This requires justification for the choice of the speech signal appropriate features. Their characteristics should not depend on the moment of a speech signal selection (to be invariant in time), contain the maximum data about the signal, simply formed and measured, and take up little space on the screen of the visualizing device for effective perception by the patient.

In mathematical models of speech signal, designed for automatic speech recognition and synthesis (J.L. Flanagan [6], L.R. Rabiner, R.V. Shafer [7], L.R. Bohl, F. Jelinek, K.L. Mercer [8], T.K. Vintsyuk [9], V.M. Velichko, N.G. Zagoruiko [10], J.Laroche, Y.Stylianou, E.Mouflines [11] and software products of IBM, Creative Technology, Dragon System etc.), the main attention is paid to the features of the speech message, and not to the visual display of the spoken sounds, which makes these models unsuitable for solving the problem of a speech apparatus training. Therefore, the grounding of a speech signal mathematical model, development of data processing algorithms for its use in biotechnical rehabilitation system, determining the composition and structure of the system, assessing its stability and physical feasibility on the basis of the selected model is relevant.

2. Methods of a speech signal modeling based on a signal concept

Questions concerning methods of modeling of a speech signal on the basis of the signal concept are considered. The possibilities offered by the use of known deterministic and stochastic models for the task of training the speech apparatus are analyzed. Particular attention when choosing a model is paid to the possibility of detecting with its help a time-invariant feature of the speech signal. Windows have been selected for more efficient use of the model.

To obtain the basic mathematical dependences, the description of the speech signal as an oscillation is used, including one in which, together with repetition, randomness plays a significant role.
A vowel [а] of a healthy person who hears and speaks well is selected for research. An informative sign of the signal invariant to time shifts is searched for. Since periodicity of vowel sounds is observed in time recordings, we approach it as a periodic function when modeling a signal. We use methods of harmonic analysis, spectral analysis and energy theory of stochastic signals [12,13]. As a result of the initial description we obtain a set of a speech signal models.

Mathematical models of oscillations in the form of periodic and almost periodic functions make it possible to describe the regularities of vowel sound with a discrete or continuous spectrum in terms of models of deterministic functions through their images in certain spaces.

In particular, space $L^2([0, T])$ is used, which is a partial case of the Hilbert space of periodic functions, for which a value $\int_0^T |f(t)|^2 dt$, where $T$ - signal period, $t, T, f(t) \in \mathbb{R}$. The basis in such a space is a set of harmonic oscillations $\{e^{i\frac{2\pi k}{T}t}, k \in \mathbb{Z}\}$. Speech signal as a function of numerical functions space $L^2([0, T])$ - designated on the set $[0, T]$ with an integral square - can be represented by the series converging in the mean square sense:

$$f(t) = \sum_{k \in \mathbb{Z}} c_k e^{i\frac{2\pi k}{T}t}, \quad t \in [0, T],$$

where: $c_k = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi k}{T}t} dt$.

By Parseval's theorem we obtain $\frac{1}{T} \int_0^T |f(t)|^2 dt = \sum_{k \in \mathbb{Z}} |c_k|^2$. This justifies the decomposition of periodic functions describing the oscillations into simple harmonic components.

If we generalize expression (1), replacing the arguments $\frac{2\pi k}{T}$ complex harmonics ordered in ascending order of arbitrary $\omega_k$, then

$$f(t) = \sum_{k \in \mathbb{Z}} c_k e^{i\omega_k t}, \quad t \in [0, T].$$

Harmonic frequencies no longer define a single period and expression (1) cannot be used to find values $c_k$. To calculate the coefficients in this case, use an expression:

$$c_k = \lim_{\theta \to 0} \frac{1}{\theta} \int_{-\theta}^{\theta} f(t) e^{-i\omega_k t} dt \equiv M_\theta \{f(t)e^{i\omega_k t}\}.$$
\[\xi(\omega, t) = \sum_{n=-\infty}^{\infty} \xi(\omega, \frac{n\pi}{T}) e^{i\frac{2\pi nt}{T}}\]

According to Piranashvili's theorem, under the condition of ergodicity and finiteness of the signal, all realizations can be represented by Kotelnikov’s formula, if the random process allows the image of Karunen-Loeve.

M. Loev's introduction of the concept of harmonization and its expansion Yu.A. Rozanov discovers the structure of nonstationary random processes in oscillating systems. In the indicated and completed by Ya.P. Dragan's energy theory of stochastic signals generalizes the theory of oscillations to cases where irregularity plays a significant role along with repeatability [14].

Periodically correlated stochastic process (PCSP) of a class \(\pi^T\) - is a process whose correlation function satisfies the condition \(R(t + T, s + T) = R(t, s),\ T > 0\) for all \(t, s \in R\), and which can be written as:

\[\xi(\omega, t) = \sum_{p \in Z} \xi_p(\omega, t) e^{i\frac{2\pi pt}{T}}.\]  

through stationary components \(\xi(\omega, t), p \in Z\).

Series (2) converges in norm in the metric of the space \(L^2([0, T], H)\) and at the same time \(\|\xi\|^2_{L^2([0, T], H)} = \sum_{p \in Z} M[\xi_p(\omega, t)]^2\).

In the study of PCSP use a parametric characteristic - covariance

\[b(t, u) \equiv r(t + u, t) = \int \int e^{it\lambda} \varphi(t, d\lambda) = \sum_{p \in Z} B_p(u) e^{i\frac{2\pi pt}{T}}\]

and spectrum \(\varphi(t, \Delta) = \sum_{p \in Z} F_p(\Delta) e^{i\frac{2\pi pt}{T}}\),

where \(B_p(u)\) - correlation components; \(F_p(\Delta)\) - spectral components.

For the biotechnical rehabilitation system in the simulation of the speech signal it is proposed to use a set of spectral characteristics that fully characterize the signal during the duration, do not depend on the time of selection, i.e. are invariant to time shifts and allow to quantify the rehabilitation procedure.

3. Method of conducting an experiment of speech signal selection

A method of conducting an experiment of speech signal selection (vowel [a]) from persons who hear and speak, the choice of the signal mathematical model and results of its research are presented. A criterion for establishing the value of informative characteristics of a speech signal is proposed and a spectral-correlation method for obtaining them based on the use of numerical methods using windows is developed. The method for choosing a speech signal model is based on the fact that the mathematical apparatus of functional analysis allows considering the set of arbitrary
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objects from single positions. In the linear theory of signals, the Hilbert space $H$ of elements of an arbitrary nature, which can be determine functions, random variables, random processes, etc is used. The use of spectral characteristics is the basis for considering a signal as an element of a certain Hilbert space. An abstract Hilbert space is normalized when a norm is given and metric when the distance between elements is given. The additivity of "spectra" makes it possible to perform arithmetic operations on them. The presence of the metric allows you to enter the resolution threshold as the distance between two time-shifted spectra of fragments (samples) of the same signal (vowel [a]) for a particular patient for the selected speech signal model.

Having received the resolution thresholds for one patient for all models from the proposed set, we select a speech signal model for this patient according to the minimum resolution threshold value. When rehabilitating the functions of the speech apparatus, the "proximity" of the sound pronounced by the patient to the desired (reference) is the distance between the two elements of the Hilbert space (taking into account the value of the resolution threshold), which must be reduced to zero during training. Because the research was conducted for a healthy person, any fragment of the implementation can serve as an ethalone.

When considering the signal as a random process, it was assumed that the vowel [a] is an ergodic random process. Under this assumption, the PCSP decomposes into a set of co-phase values on lattices of the form $\{t_0 + kT, \ k = -\infty, \infty\}$, which differ in values $t_0 \in [0, T]$. Such sets form stationary ergodic and stationary ergodically connected random sequences. From the PCSP image through stationary components $\xi(t) = \sum_{k=-\infty}^{\infty} \xi_k(t) e^{ik2\pi t}$ it follows that the PCSP counts through the correlation period are stationary random sequences, and the sum of the stationary process counts is stationary.

The criterion of statistical estimates correctness for processes of class $\pi$ is the fulfillment of correlation links rapid attenuation condition $\lim_{|u|\to\infty} B(u) = 0$ for all $t$, where $B(u)$ - mean covariance of a stochastic process that has all the covariance properties of a stationary stochastic process. With a known mathematical expectation, the estimate of the correlation function has the form:

$$\hat{b}_\xi(t, u) = \frac{1}{N} \sum_{k=0}^{N-1} \xi(t + kT) \xi(t + u + nT) =$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \xi(t + kT) \xi(t + u + nT) - m_\xi(t)m_\xi(t + u),$$

where $T$ - correlation period; $m_\xi(t) = E\xi(t)$ - unbiased.

Estimates of decomposition coefficients $f_k(\omega)$ - spectral components, we obtain from the estimates of the correlation components
\[ \hat{f}_k = \frac{1}{2\pi} \int_0^T \hat{B}_k(u)e^{-iu\omega}du \]

To confirm the correctness of the assumptions about the invariance to temporal shifts of the spectral characteristics of the models and the feasibility of using the developed algorithms, their experimental verification was carried out.

4. Substantiation of the rehabilitation system structural scheme construction

The purpose of verification is to check the adequacy of the speech signal model by means of full-scale modeling (using the example of the vowel [a]); interpretation of the results obtained; substantiation of practical recommendations for its use for the rehabilitation of the functions of the speech apparatus.

Fundamental in the developed approach is that we treat a person in combination with technical means as a unified information and measurement system. Based on the idea of creating a language process, a model of perception and creation of a language by a deaf person and the structure of a specialized rehabilitation system is proposed in the form [15]:

![Figure 1. Structural diagram of speech perception when replacing the auditory analyzer with a visual one (1-brain; 2-motor speech center of the brain; 3-organ of speech; 4-microphone; 5-preprocessing unit; 6-visualization unit; 7-organ of vision; 8-visual centre)](image)

Using this interpretation, the diagram shown in fig. 1, we give in canonical form (fig. 2).

![Figure 2. Canonical representation of the language perception structural scheme](image)
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Noticing the similarity of the functions of the block-executing circuits in fig. 1 and fig. 2, we will consider the circuit in fig. 2 model of perception and creation of speech by a person with hearing impairments when replacing the auditory analyzer with a visual one.

Here $U_{in}$ corresponds to a speech signal that a person must repeat (ethalone signal); bloc $K_1$ simulates the procedure of sound pronunciation by the patient’s speech apparatus. Bloc $K_2$ simulates selection, processing and visualization of the speech signal in the feedback channel. Block $K_3$ simulates a person’s perception of a visual image. The comparison unit simulates the work of the human brain when he comprehends visual images of the ethalone and spoken sound. $U_{out}$ – a patient’s speech signal.

In order to study the conditions for the stability of the system’s operation, we will take into account that:

a) the patient’s reaction is delayed by the amount of the latent period, which depends on individual psychophysiological qualities;

b) the time of inertia for the perception and comprehension of what was seen depends on the type of the reference educational image and in the process of training decreases to a certain (different for each person) value.

Given this into account, we present the transfer function of the system in the form:

$$K(p) = \frac{\frac{k_1}{1+K_1K_2K_3}}{\frac{k_1e^{-pT_1}}{(T_p+1)(1+k_1K_2K_3e^{-p(T_1+T_2+T_3)})}} = \frac{k_1e^{-pT_1}}{T_p+1+k_1K_2K_3e^{-p(T_1+T_2+T_3)}}$$

For the first approximation, values of the coefficients $k_1, k_2$ and $k_3$ were taken as equal 1. This choice is due to the adequate (undistorted) perception of the patient of the data received by him. The stability of the system is determined by obtaining the poles of the transfer function, which are the solutions of the characteristic equation: $T_p + 1 + k_1k_2k_3e^{-p(T_1+T_2+T_3)} = 0$, which we look for using the values of variables obtained experimentally for a particular patient. For the system to be stable, it is necessary and sufficient that the poles of the transfer function are located in the left half-plane of the roots $p$. Experimental studies have shown that the system loses stability when a person’s psycho-physiological capabilities are such that he or she cannot use the system to rehabilitate speech function (the time of human perception of the visual educational image is close to 90 s) or, when the selection, processing and visualization of the speech signal by hardware takes about 90 s. The time spent by a person is determined experimentally, and the processing time depends largely on the signal model. When choosing a model of the speech signal, hypotheses are accepted that the model can be: a) a deterministic function; b) stationary stochastic process; c) non-stationary periodically correlated stochastic process.

In order to accept one of the hypotheses, rejecting the others, an experimental verification of the model was performed, that is, the correspondence of the deductive conclusions about the model to the properties of the speech signal described using such a model was checked. The decisive role was played by the ability to create a signal ethalone on the basis of the model, which would allow assessing the proximity of the sound pronounced by the patient and the ethalone. Since a number of models were considered, then as a criterion for the optimality of the model from the proposed set the value of the threshold of discrimination between the running and the reference
spectral characteristic of the speech signal samples, which was minimal in terms of the set of models, was taken. For the resolution threshold, the maximum standard deviation of the spectral characteristics of the reference sample from the remaining 5 for a given model was taken. To ensure the correctness of the estimation of the spectral components during the verification of the PCSP as a model of the speech signal, smoothing with the help of windows was applied. The estimation of the spectral components took the form:

$$f_k(\omega) = \frac{1}{2\pi} \int_0^T \hat{B}_k(u)w(u)e^{-i\omega u}du,$$

where $$w(u)$$ - window weight function.

Research has been done for the following windows:

a) Hamming window: $$w(u) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N}\right), n = 0, 1, ..., N - 1;$$

b) Fejer and Bartlet's triangular window: $$w(u) = \left\{ \begin{array}{ll}
\frac{n}{N/2}, & n = 0, 1, ..., N/2; \\
(w(N - n), & n = N/2, ..., N - 1;
\end{array} \right.$$

c) Blackman-Harris window:

$$w(u) = 0.42323 - 0.49755 \cos\left(\frac{2\pi n}{N}\right) + 0.07922 \cos\left(\frac{2\pi n}{N}\right), n = 0, 1, ..., N - 1.$$ 

Studies have shown that the amplitude and energy spectra are sensitive to changes in the intensity of the speech signal. As a result, the distance between the ethalone and any other signal sample in the deterministic and stochastic stationary model varies quite widely over time.

Distance values obtained when using PKSP:

$$\rho_1 = \{0.0925; 0; 0.0605; 0.1023; 0.0518; 0.1508\}, \rho_{\text{max}} = 0.2;$$

$$\rho_2 = \{0.0834; 0.0567; 0.0933; 0.0516; 0.1432\}, \rho_{\text{max}} = 0.2;$$

$$\rho_3 = \{0.0852; 0.0566; 0.0855; 0.0453; 0.1509\}, \rho_{\text{max}} = 0.2;$$

The values of the discrimination threshold give grounds to say that the speech signal model as the PCSP is most suitable for use in the rehabilitation system, and when using the Blackman-Harris window, which allows taking into account close but significantly different tones in amplitude, this model is the least sensitive to signal amplification within the range of intensity variation sound (see table 1), which will occur during the rehabilitation of the speech apparatus.
Table 1. Resolution threshold value when changing sound intensity (vowel [a])

<table>
<thead>
<tr>
<th>Window</th>
<th>Gain</th>
<th>Resolution threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming</td>
<td>0.5</td>
<td>0.1221</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.2775</td>
</tr>
<tr>
<td>Triangular</td>
<td>0.5</td>
<td>0.1123</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.2530</td>
</tr>
<tr>
<td>Blackman-Harris</td>
<td>0.5</td>
<td>0.1116</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.2501</td>
</tr>
</tbody>
</table>

When describing a speech signal as a PCSP, the signal period coincides with the correlation period, which greatly simplifies its calculation.

In the recommendations on the method of this model using for the speech apparatus functions rehabilitation of the people with hearing impairments, a speech signal is proposed to receive from the patient using a probe of a known design, on the basis of which an ethalone training image will be formed, and the formation of the ethalone image itself should be coordinated with the features of the developed model.

One of the possible ways to visualize a speech signal is to use as a ethalone a regular rectangle of arbitrary width, the length of which will be equal to the signal norm, $\|\xi(\omega, t)\| = \left\{M \int_{\omega} |\xi(\omega, t)|^2 dt \right\}$, including the permissible resolution threshold, represented at a certain scale (fig. 3).

Figure 3. An example of a speech signal visual image for the rehabilitation of a speech apparatus functions

The current signal is represented by a similar rectangle with a variable length, depending on the quality of the speech signal. The length of the running rectangle will change in accordance with the resolution threshold calculated for a given patient during the rehabilitation process. If the right limit of the upper rectangle falls into the interval indicated by “a”, we consider that the goal of rehabilitation has been achieved, if not (dotted lines in the figure), the training process must be continued.

The proposed recommendations can be used by doctors in the development of a rehabilitation technique, which will allow an individual approach to the patient during the rehabilitation of the functions of the speech apparatus and assess the quality of the rehabilitation procedure.
Conclusion

Some conclusions can be made. The use of the PCSP for modeling the registrogram of a speech signal has been substantiated, which made it possible to obtain its informative features. The spectral components of the PCSP were selected as an informative feature of a speech signal, which made it possible to construct a visual training image of a vowel in a biotechnical rehabilitation system for training the speech apparatus of people with hearing impairments. A spectral-correlation method was developed to obtain the values of spectral components, which made it possible to construct a model verification criterion. Software has been developed for calculating the values of spectral components, which made it possible to obtain numerical values of the approximation of the speech signal running to the ethalone registrogram and to quantitatively assess the quality of rehabilitation. An ethalone teaching visual image of a speech signal was obtained, consistent with the features of the selected model on the basis of a developed computational and experimental method, which made it possible to develop recommendations for creating a rehabilitation method. For the first time, a substantiated choice of the biotechnical system structure for speech apparatus functions rehabilitation was done, which made it possible to determine the criterion of its stability. Based on the results of theoretical and experimental researches, recommendations were developed on the use of the proposed model when creating new methods of training the speech apparatus, which allow to take into account the individual patient’s characteristics in the rehabilitation process.

REFERENCES


