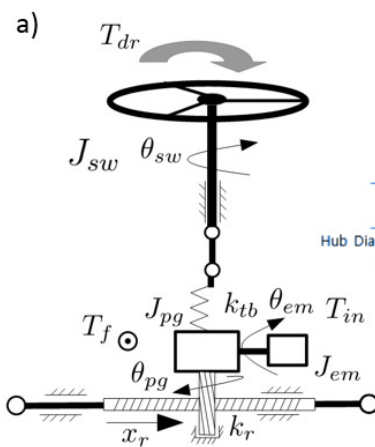


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Analysis of the electric power steering system's worm gear friction impact on the steering wheel torque



$$\dot{z} = v - \frac{|v|}{g(v)} \cdot z \quad (1)$$

$$F_f = \sigma_0 \cdot z + \sigma_1 \cdot \dot{z} + \sigma_2 \cdot v \quad (2)$$

$$g(v) = \frac{1}{\sigma_0} \cdot \left(F_C + (F_S - F_C) \cdot e^{-\left(\frac{v}{v_S}\right)^2} \right) \quad (3)$$

where:
 z, \dot{z} - friction state and its first derivative [1]
 F_f - friction force
 v - sliding velocity
 $g(v)$ - function capturing Coulomb friction and Stribeck effect [1], [2]
 $\sigma_0, \sigma_1, \sigma_2, F_C, F_S, v_S$ - refer Table 1

Table 1. LuGre model parameters [1], [2]

Parameter	Value	Unit	Meaning
σ_0	10^5	N/m	stiffness
σ_1	$\sqrt{10^3}$	Ns/m	micro-damping
σ_2	0.4	Ns/m	viscosity
F_C	1	N	Coulomb friction force
F_S	1.5	N	Stiction force
v_S	0.001	m/s	speed of approaching function $g(v)$ to Coulomb friction force

$$[A] \cdot \ddot{\varphi} + [B] \cdot \dot{\varphi} + [C] \cdot \varphi = q \quad (5)$$

$$\begin{bmatrix} J_{sw} & 0 & 0 & 0 \\ 0 & J_{pg} & 0 & 0 \\ 0 & 0 & m_r & 0 \\ 0 & 0 & 0 & J_{em} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_{sw} \\ \ddot{\theta}_{pg} \\ \ddot{x}_r \\ \ddot{\theta}_{em} \end{bmatrix} + \begin{bmatrix} h_{sw} & 0 & 0 & 0 \\ 0 & h_{pg} & 0 & 0 \\ 0 & 0 & h_r & 0 \\ 0 & 0 & 0 & h_{em} \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_{sw} \\ \dot{\theta}_{pg} \\ \dot{x}_r \\ \dot{\theta}_{em} \end{bmatrix} + \begin{bmatrix} k_{tw} & -k_{tw} & 0 & 0 \\ -k_{tw} & k_{tw} + k_r & -\frac{k_r}{r} & 0 \\ 0 & -\frac{k_r}{r} & \frac{k_r}{r^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \theta_{sw} \\ \theta_{pg} \\ x_r \\ \theta_{em} \end{bmatrix} = \begin{bmatrix} T_{dr} \\ i \cdot (T_{in} - T_f) \\ F \\ T_{em} - T_{in} \end{bmatrix}$$

where:
 $\theta_{sw}, \theta_{pg}, \theta_{em}$ - respectively steering wheel, pinion and electric motor angles
 $T_{dr}, T_{in}, T_r, T_{em}$ - respectively driver, assistance, friction and DC motor torques

